How To Stress Test Benchmark Relative Portfolios

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As stress testing becomes more and more widespread as a risk measurement tool of choice, new questions are formulated about its applications to portfolio management. One of the most important ones for fund managers is: ‘How do we analyze results of stress tests for benchmarked portfolios?’ We will see that the standard approach of stress testing decomposition that works very well for absolute return portfolios is actually quite misleading when applied to those managed against the benchmark. We will suggest performance attribution based approach to the analysis of the result of the stress tests that overcomes these problems and results in much clearer risk signals to the portfolio managers.

Mechanics of Stress Testing

The results of stress tests are calculated using the formula first presented in Kupiec (1998). Let

\[
x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}
\]

be a normal random vector with zero mean and covariance matrix \( C \).

Let us describe our covariance matrix \( C \) as:

\[
C = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}
\]

(1)

Then Feller (1970) conditional distribution of \( x_1 \) given \( x_2 = \alpha \) is normal with conditional mean equal to

\[
m = C_{12}C^{-1}C_{22} \alpha
\]

(2)

Where \( m \) is a vector of shocked factor returns in \( x_1 \).

This conditional mean is the conditional return for all factors in \( x_1 \) given the specified shock(s) in \( x_2 \). If we specified in our stress test that random variables Equities drop by 40% and High Yield spreads widen by 18%, the factors for Equities and High Yield spread would be the \( x_2 \) and the rest of the factors in the model would be \( x_1 \). The stress test result for the portfolio is obtained by multiplying:

\[
S = wm
\]

(3)

Where \( W \) is the exposure of the portfolio onto each of the factors in \( x_1 \) multiplied by the respective conditional return for those same factors calculated in (3) above. This formula is standard, used by all major risk providers and regulation agencies for performing stress tests.

Issues With Standard Stress Test Impact Decomposition

Standard stress test decomposition is calculated by taking a beta of every security in the portfolio and benchmark to each factor in the model and multiplying it by the simulated move of that factor in a given scenario. The return in a given stress test for security \( i \) is calculated based on the conditional mean formula (3). Let’s write it in the vector notation to define the contribution of security \( i \):

\[
ST(i) = \sum_{k=1}^{N} \beta_{i,k} \cdot m_k
\]

(4)
Where:

\( N \) - the number of factors in a risk model

\( \beta_{i,k} \) - beta of security (i) to factor (k)

\( m_k \) - simulated return of factor (k) in a given scenario

From returns of each security the overall portfolio active return is calculated as:

\[
ST(P) = \sum_{i=1}^{M} AW(i) \times ST(i)
\]  

(5)

Where:

\( M \) - number of securities in the portfolio

\( AW(i) \) - active weight of security (i) in the portfolio

Note that result of formula (5) is equal to result of formula (3), except that it uses individual contributions where formula (3) used matrix notation.

Formula (4) is a standard method to decompose results of stress tests on position level. What then is the problem with this decomposition? The answer is that it can be misleading at times and may not produce the desired response of the portfolio manager when presented with risk analysis. We will consider one example on a sector level and one on a security level. In each example, we will suggest another decomposition that should be used as a supplement to the standard one in formulae (1) & (2).

**Problem and Solution at the Sector Level**

Consider a given scenario, say Euro Meltdown (we modeled Euro Meltdown as EUR down 40% against USD and UST 30 Year Yield going down by 100 bps). Let’s analyze the impact on our sample Growth portfolio which is benchmarked against S&P 500 (projected Tracking Error is 2.8). Benchmark loses 35.88% in this scenario. Let’s focus on the Consumer Staples sector in Exhibit 1. We have clearly made some strong bets in this sector, overweighting it by 2.14%. Standard stress testing decomposition from formulae (1) & (2) shown in the column titled ‘CTR +/-’ shows a negative contribution from this sector. This should not be surprising, as the standard decomposition is simply a return of each security multiplied by active weight summed up for the sector. In this decomposition every overweight sector will almost always have a negative contribution and vice versa. This is because in stress testing we are dealing with stress events by definition and most assets will be down in such events.
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*Exhibit 1 – Standard Stress Testing Decomposition in a Euro Meltdown Scenario*

<table>
<thead>
<tr>
<th>GICS Sector</th>
<th>Portfolio Weight (%)</th>
<th>Benchmark Weight (%)</th>
<th>Active Weight (%)</th>
<th>CTR +/-</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>7.83</td>
<td>10.09</td>
<td>-2.55</td>
<td>1.12</td>
</tr>
<tr>
<td>Materials</td>
<td>5.67</td>
<td>3.45</td>
<td>2.22</td>
<td>-0.46</td>
</tr>
<tr>
<td>Financials</td>
<td>18.13</td>
<td>16.20</td>
<td>1.93</td>
<td>-1.71</td>
</tr>
<tr>
<td>Industrials</td>
<td>10.15</td>
<td>10.39</td>
<td>-0.25</td>
<td>0.09</td>
</tr>
<tr>
<td>Consumer Discret.</td>
<td>14.95</td>
<td>11.84</td>
<td>3.11</td>
<td>-3.29</td>
</tr>
<tr>
<td>Information Tech.</td>
<td>15.59</td>
<td>19.44</td>
<td>-3.85</td>
<td>1.23</td>
</tr>
<tr>
<td>Utilities</td>
<td>2.89</td>
<td>2.96</td>
<td>-0.07</td>
<td>0.02</td>
</tr>
<tr>
<td>Health Care</td>
<td>13.50</td>
<td>13.83</td>
<td>-0.33</td>
<td>-1.25</td>
</tr>
<tr>
<td><strong>Consumer Staples</strong></td>
<td><strong>11.59</strong></td>
<td><strong>9.45</strong></td>
<td><strong>2.14</strong></td>
<td><strong>-0.10</strong></td>
</tr>
<tr>
<td><strong>Telecomunicati...</strong></td>
<td><strong>0.00</strong></td>
<td><strong>2.35</strong></td>
<td><strong>-2.35</strong></td>
<td><strong>0.48</strong></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>100.00</strong></td>
<td><strong>100.00</strong></td>
<td><strong>0.00</strong></td>
<td><strong>-3.86</strong></td>
</tr>
</tbody>
</table>

How would we solve this problem? The answer is simple, because this problem was already solved with various performance attribution modules. Let’s consider how a Brinson-Fachler type attribution would inform our decision making in the same case. We can see from Exhibit 2 directly below that Consumer Staples, far from being detrimental, is actually positioned very well for a Euro event.

*Exhibit 2 – Brinson-Fachler type attribution in the Euro Meltdown Scenario*

<table>
<thead>
<tr>
<th>GICS Sector</th>
<th>Portfolio Weight (%)</th>
<th>Benchmark Weight (%)</th>
<th>Active Weight (%)</th>
<th>Total Return - Port</th>
<th>Total Return - Bmk</th>
<th>Attribution - Alloc</th>
<th>Attribution - Select</th>
<th>Attribution - Our</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>7.53</td>
<td>10.09</td>
<td>-2.55</td>
<td>-59.72</td>
<td>-59.72</td>
<td>0.51</td>
<td>-0.51</td>
<td>0.00</td>
</tr>
<tr>
<td>Materials</td>
<td>5.67</td>
<td>3.45</td>
<td>2.22</td>
<td>-36.86</td>
<td>-47.11</td>
<td>-0.25</td>
<td>0.56</td>
<td>0.00</td>
</tr>
<tr>
<td>Financials</td>
<td>18.13</td>
<td>16.20</td>
<td>1.93</td>
<td>-46.38</td>
<td>-41.38</td>
<td>-0.11</td>
<td>-0.17</td>
<td>-0.73</td>
</tr>
<tr>
<td>Industrials</td>
<td>10.15</td>
<td>10.39</td>
<td>-0.25</td>
<td>-36.56</td>
<td>-36.56</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Consumer Discret.</td>
<td>14.95</td>
<td>11.84</td>
<td>3.11</td>
<td>-50.27</td>
<td>-35.71</td>
<td>0.00</td>
<td>-0.89</td>
<td>-1.28</td>
</tr>
<tr>
<td>Information Tech.</td>
<td>15.59</td>
<td>19.44</td>
<td>-3.85</td>
<td>-31.91</td>
<td>-31.95</td>
<td>-0.15</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>Utilities</td>
<td>2.89</td>
<td>2.96</td>
<td>-0.07</td>
<td>-28.39</td>
<td>-28.39</td>
<td>0.00</td>
<td>-0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>Health Care</td>
<td>13.50</td>
<td>13.83</td>
<td>-0.33</td>
<td>-37.52</td>
<td>-27.88</td>
<td>-0.03</td>
<td>1.34</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Consumer Staples</strong></td>
<td><strong>11.59</strong></td>
<td><strong>9.45</strong></td>
<td><strong>2.14</strong></td>
<td><strong>-19.00</strong></td>
<td><strong>-22.23</strong></td>
<td><strong>0.29</strong></td>
<td><strong>0.37</strong></td>
<td><strong>0.00</strong></td>
</tr>
<tr>
<td><strong>Telecomunicati...</strong></td>
<td><strong>0.00</strong></td>
<td><strong>2.35</strong></td>
<td><strong>-2.35</strong></td>
<td><strong>0.00</strong></td>
<td><strong>-20.57</strong></td>
<td><strong>0.36</strong></td>
<td><strong>0.00</strong></td>
<td><strong>0.00</strong></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>100.00</strong></td>
<td><strong>100.00</strong></td>
<td><strong>0.00</strong></td>
<td><strong>-39.69</strong></td>
<td><strong>-35.83</strong></td>
<td><strong>0.01</strong></td>
<td><strong>-1.86</strong></td>
<td><strong>-2.02</strong></td>
</tr>
</tbody>
</table>

Rather than giving us a negative contribution, Consumer Staples gives positive effects from both Sector Allocation and Security Selection, without the loss in Currency Effect that is present in some other sectors of this portfolio.

**Sector Allocation**
We propose using Brinson-Fachler attribution to measure contributions in ex ante scenarios just as it is used for ex post performance, see Brinson et al (1985). Sector allocation effect in performance attribution models is equal to:

\[ SA = AW(s) \times (BenchRet(s) - BenchRetTotal) \]  

(6)

Where:

- \( AW(s) \) - active weight of the sector
- \( BenchRet(s) \) - return of a sector within the benchmark
- \( BenchRetTotal \) - total return of the benchmark

In our case that means:

\[ 2.14 \times (-22.23 - (-35.83)) / 100 = .29 \]

The reason for the positive allocation effect is that Consumer Staples benchmark loss is much smaller than the overall loss for the benchmark.

**Security Selection**

Security selection is positive because we picked better securities. Portfolio loss in Consumer Staples is -19%, while benchmark loses 22.23%. Both are better than the overall benchmark, but the portfolio manager picked better securities.

**Security Level Impacts**

One issue with Brinson-Fachler is that it is a sector model and is not meant to provide insights into individual security behavior. But in stress testing it is crucial to identify specific securities that pose risks in a given stress test. For example, if our hypothetical portfolio manager was trying to understand portfolio vulnerabilities in a Euro Meltdown, he or she would need to find securities with risk concentration.

Typically, a standard decomposition in formula (1) is used for that purpose. However, it suffers from the same limitation on the security level as it does on the group level. Namely, that it is a pure contribution and in stress scenario most of the overweights will be penalized, because most assets lose. Consider, for example, Clorox in Exhibit 3. It shows a negative contribution because it is an overweight of 2.93 and it loses 9.8% in a Euro Meltdown, so the contribution is negative.
We can solve this problem by introducing a different kind of contribution:

$$SC(i) = AW(i) \times (Sec\,Ret(i) - Bench\,RetTotal)$$  \hspace{1cm} (7)

Where:

$Sec\,Ret(i)$ - return of security (i) in a given stress scenario

Once we redefine the contribution from each security in this way, we will get a result for Clorox that is much more actionable for a PM. It is clearly a good bet given that it loses less than 10% in a scenario where the benchmark is down 35.83%. The new contribution gives us +.76%. The new contribution in formula (7) also adds up to the total over/underperformance of the portfolio which is a pre-requisite for a decomposition.
Summary

Stress testing is becoming ever more popular as other types of risk analysis have shown to be deficient in dealing with tail events. However, results of stress testing need to be made more actionable for both risk and portfolio managers. Simple modifications to the way that scenarios are decomposed can increase usefulness of stress testing for managers with benchmarked portfolios. On a sector level we blended stress testing with Brinson-Fachler decomposition. On a security level we suggest modifying the contribution to reflect the over/underperformance of that security to the overall benchmark return.

References:


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